Abstract - The aspect of this paper entitled “Dynamics of 3-DOF R-R-R Type Manipulator Arm” is to give the brief idea about the Dynamics is important for mechanical design, control, and simulation of the Manipulator Arm. Initially study has been done on the industrial robots, their applications, and the problems associated with the current robotic arms, and the work has been carried out on the solution of the problem. Literature Study has given ideas in the field of Mechanical design and Dynamics of linkages. 3-DOF non planer R-R-R type of manipulator is chosen for Mechanical design and Dynamics analysis. For structural design The dynamic equations of motion provide the relationships between actuation and contact forces acting on robot mechanisms, and the acceleration and motion trajectories that result.

Key words- robot, link mechanism, light weight, low cost, Deflection

“I. INTRODUCTION”

One of the primary objectives of robotics engineering is to design a manipulator capable of high link accelerations without sacrificing positional accuracy. Concern of current robotic research has been to develop anthropomorphic (human-like) arms capable of emulating the dexterity, manipulability, and workspace volume and payload-to-weight ratio of a human arm. The advent of composite materials, with very high stiffness-to-weight and strength-to-weight ratios as well as excellent damping properties, have made it possible for robotics engineers to build manipulators with excellent stiffness, strength, damping and low inertia.

1.1 Dynamics

Dynamics is important for mechanical design, control, and simulation. Inverse dynamics used in feed-forward control (required torques and forces computed),forward dynamics used for simulation. it determined joint acceleration based on torques and forces are specified .the joint-space inertia matrix is used in analysis in feedback control to line arise the dynamics and integral part of many forward dynamics formulation and the operational-space inertia matrix used in control at the task at end effectors.

Dynamics provides the relationships between actuation and contact forces, and the acceleration and motion trajectories that result. The dynamic equations of motion provide the basis for a number of computational algorithms that are useful in mechanical design, control, and simulation.

1.1.1 Problems Associated with Current Robotic Arms

Most current robotic arms possess poor payload-to-weight ratios, poor damping and lack anthropomorphic manipulability and dexterity. Conventionally, to design a fast-moving arm required that the links have low inertia. Inevitably, this resulted in large end-effectors vibrations and long settling times. Conversely, to achieve high Positional accuracy required bulky, massive links. Due to the large inertia of the links, these robotic arms cannot move rapidly and require inordinate amounts of power. However, robot researchers the world over have already begun to offer many design solutions to these problems. To achieve the manipulability and dexterity of a human arm, innovative new joint mechanisms have been studied.
1.2 Design of the Manipulator Arm:

Figure 1: Manipulator Arm Structure (Front View)
Consider a manipulator arm is the cantilever type of structure with payload to weight ratio 1:1 with maximum deflection 1 CM. In fig.1 the structure is fixed at point A which will be attached to the robot base structure. Section AB is the hollow circular section of 3 mm thickness, which will be used to join the robot arm structure to the robot base structure. Section CD is the main arm structure which is hollow circular in cross section with 3 mm of thickness. Section EF is the hollow circular section of 3 mm thickness which will be used to fix the motor and the gripper mechanism. Further gripper will be attached to the section EF.

1.2.1 Generalized Forces:

The generalized force $Q_i$ is defined as $Q_i = \sum_{j=1}^n F_j \left( \frac{\partial r_j}{\partial \Theta_i} \right)$ where $F_j$ is the force at point $j$ and $r_j$ is the position vector of point $j$. The index $i$ correspond to Generalized coordinates.

1.2.2 Equations of Motions:

The equations of motion of a robot mechanism are usually presented in one of two Canonical forms: the joint-space formulation, $I(\Theta) \dot{\Theta}^\ast + C(\Theta, \dot{\Theta}) \dot{\Theta}^\ast + \tau_g (\Theta) = \tau$ or the operational-space formulation, $A(x) \dot{v}^\ast + \mu(x, v) + \rho(x) = f$
where $f=\text{ the net force acting on a rigid body and is given by } f = Ia + v \times \dot{I}v$ where $a = \text{ acceleration, } v = \text{ velocity. These equations show the functional dependencies explicitly: } I \text{ is a function of } \Theta, A \text{ is a function of } x, \text{ and so on. } x \text{ is a vector of operational-space coordinates, while } v \text{ and } f \text{ are spatial vectors denoting the velocity of the end-effectors and the external force acting on it. If the robot is redundant, then the coefficients of this equation must be defined as functions of } \Theta \text{ and } \Theta^\ast \text{ rather than } x \text{ and } v.$

1.3 Lagrange Formulation:

The two methods that are most commonly used in robotics are the Newton–Euler formulation and the Lagrange formulation. The former works directly with Newton’s and Euler’s equations for a rigid body, which are contained within the spatial equation of motion. This formulation is especially enabled to the development of efficient recursive algorithms for dynamics computations.

The Lagrange formulation proceeds via the Lagrangian of the robot mechanism, $L = K.E - P.E$ where $K.E$ and $P.E$ are the total kinetic and potential energy, respectively, of the mechanism.

The kinetic energy is given by $K.E = \frac{1}{2}[\Theta^\ast]^T[I(\Theta)] \dot{\Theta}^\ast$ The dynamic equations of motion can then be developed using Lagrange’s equation for each generalized coordinate: Torque $\tau_i = \frac{d}{dt} \left( \frac{\partial (L)}{\partial \dot{\Theta}_i} \right) - \frac{\partial (L)}{\partial \Theta_i}$

The resulting equation can be written in scalar form: $\sum_{j=1}^n I_j \ddot{\Theta}_j + \sum_{j=1}^n \sum_{k=1}^{n} C_{jk} \dot{\Theta}_j \dot{\Theta}_k + \tau_{\text{eff}} = \tau_i$

1.3.1 Lagrange-Euler Dynamic Model Algorithm for the Closed-Form Equation of Motion:

This algorithm carries out the complete dynamic formulation of an n-DOF manipulator that satisfies the condition for existence of closed-form geometric solutions. The various steps are

**Step 1** Assign frames \{0\},....\{n\} using DH notation such that frame \{i\} is oriented (aligned) with principle axis of link i.

**Step 2** Obtain the link transformation matrix $T_i^{j-1}$ for each link and from these compute product matrices $T_2^0, T_3^0$ and so on, which are required for computing the coefficients $d_{ij}$ and its derivatives, using equation...
Step 3 Define partial derivative velocity matrix Qi for each link, depending on whether the joint is revolute or prismatic.

Step 4 For each link i determine the inertia tensor Ii with respect to frame \{i\}

Step 5 Compute \( d_{ij} \) for \( i, j = 1, 2, \ldots, n \) using equation

\[
\begin{align*}
T_{ij} &= Q_i T_{i,j}^{-1} \quad \text{for } j \leq i \\
0 &= \quad \text{for } j > i
\end{align*}
\]

Step 6 Compute the inertia coefficients \( M_{ij} \) for \( i, j = 1, 2, \ldots, n \) using equation

\[
M_{ij} = \sum_{p=\max(i,j)}^{n} T_{ij}^{p} d_{p}^{T} d_{p}^{T}
\]

Step 7 Compute the velocity coupling coefficients \( h_{ijk} \) for \( i, j, k = 1, 2, \ldots, n \) using equation

\[
\begin{align*}
\frac{\partial d_{ij}}{\partial q} &= \begin{cases} 
T_{ij}^{0} Q_i T_{i,j}^{-1} Q_j T_{j,i}^{-1} \quad \text{for } i \geq k \geq j \\
T_{ki}^{0} Q_k T_{k,i}^{-1} Q_i T_{i,k}^{-1} \quad \text{for } i \geq j \geq k \\
0 & \text{for } i < j \text{ or } i < k
\end{cases}
\end{align*}
\]

Step 8 Compute gravity loading terms \( G_i \) for each link, \( i = 1, 2, \ldots, n \) using equation

\[
G_i = -\sum_{p=1}^{n} m_p g d_{pi} T_{p}^{T}
\]

Step 9 To formulate the \( i^{th} \) equation for torque \( \tau_i \), substitute all the coefficients in equation

\[
\tau_i = \sum_{j=1}^{n} M_i(q) q_j + \sum_{h=1}^{n} h_{ih} q_h \dot{q}_i + G_i \quad \text{for } i = 1, 2, \ldots, n
\]

### 1.3.2 Dynamic Analysis Formulation for 3–DOF 4 Link RRR Type Manipulator Arm using Lagrange–Euler Approach:

Fig 1.1 Frame assignment for 3-DOF 4-Link RRR Type Manipulator Arm

The manipulator is shown in the Fig. based on assumptions that all the four links, Link 1, Link 2, link 3, and Link 4 are cylindrical with mass \( m_1, m_2, m_3, m_4 \) respectively at their distal end, and Link 4 is rigidly connected with link 3. The Lagrange-Euler formulation is carried out to obtain the EOM as per Algorithm. The frames assignment is shown in Fig. 1 and Table 1 gives the joint link parameters.

<table>
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<th>( i )</th>
<th>( \Theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
<th>C ( \Theta_i )</th>
<th>S ( \Theta_i )</th>
<th>C ( \alpha_i )</th>
<th>S ( \alpha_i )</th>
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<td>( L_1 )</td>
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<tr>
<td>4</td>
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<td>( L_4 )</td>
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<td></td>
</tr>
</tbody>
</table>

Table 1 Joint link parameters for 3-DOF 4-Link RRR Type Manipulator Arm
The link Transformation matrices and the overall transformation matrix are:

\[
T_1^0 = A_1 = \begin{bmatrix} C_i & 0 & -S_i & 0 \\ S_i & 0 & C_i & 0 \\ 0 & -1 & 0 & L_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = A_2 = \begin{bmatrix} s_j & 0 & c_j & 0 \\ -s_j & 0 & s_j & 0 \\ 0 & -1 & 0 & L_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
T_3^2 = A_3 = \begin{bmatrix} c_i - s_i S_i & 0 & s_i & 0 \\ S_i & 0 & C_i & 0 \\ 0 & -1 & 0 & L_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4^3 = A_4 = \begin{bmatrix} 1 & 0 & 0 & -L_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
T_2^0 = T_2^1 T_2^1 T_2^2, \quad T_3^0 = T_3^1 T_3^2 T_3^3, \quad T_4^0 = T_4^1 T_4^2 T_4^3
\]

Since all the four joints 1, 2, 3 and 4 are revolute joints, the velocity matrix is

\[Q1=Q2=Q3=Q4=\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\]

The elements of the inertia matrix \(M\) are computed

\[M_{ij} = \sum_{p=\text{masses}} T[p]d[p]I[p]d[p]^T\]

\[
M_{11} = m_2 L_2^2 + m_3 (L_2^2 + L_3^2 C_2^2) + m_4 [L_4 S_2 L_3 C_2 + L_4 S_3 L_2 C_2]^2]
\]

\[
M_{12} = M_{21} = m_3 L_2 L_3 S_2 + [m_4 (L_2 + L_3 S_3) (L_4 C_2 C_3 + L_3 S_3)]
\]

\[
M_{13} = M_{31} = m_4 [L_4 L_4 C_2 C_3 - L_4 S_2 S_3 L_2 C_3 + L_4 S_2 S_3 L_2 S_2]
\]

\[
M_{14} = M_{41} = m_4 [L_3 L_4 C_2 C_3 - L_4 S_2 S_3 L_2 C_3 + L_4 S_2 S_3 L_2 S_2]
\]

\[
M_{22} = \frac{m_3 L_3^2}{2} + m_4 (L_3^2 + L_4^2 C_2^2)M_{23} = M_{32} = -m_4 L_3 L_4 S_3,
\]

\[
M_{24} = M_{42} = -m_4 L_3 L_4 S_3, \quad M_{33} = m_4 L_4^2, \quad M_{43} = -m_4 L_4^2, \quad M_{44} = m_4 L_4^2
\]

Thus, the Acceleration-Related Symmetric matrix \(M(\Theta)\), will be

\[M(\Theta) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{11} & M_{13} & M_{14} \\ M_{13} & M_{13} & M_{11} & M_{14} \\ M_{14} & M_{14} & M_{14} & M_{11} \end{bmatrix}\]

Substitute the values of the elements \(M_{11}, M_{12}, M_{13}\) to derive the Acceleration-Related Symmetric matrix \(M(\Theta)\). The Coriolis and centrifugal force coefficient \(h_{ijk}\) for I, j, k = 1, 2, 3, 4 are computed
\[ h_{ij} = \sum_{p=\text{max}(i,j,k)}^{n} \text{Tr} \left( \frac{\partial (d_{pk})}{\partial q_j} I_p d^T_p \right) \]

\[ h_{11} = Tr(T_0^0 Q_0^0 T_0^1 I_1 d^T_{11}) = 0 \]
\[ h_{12} = h_{21} = Tr(T_0^0 Q_0^0 T_0^1 Q_1^0 T_2^1 I_2 d^T_{21}) = 0 \]
\[ h_{13} = h_{31} = Tr(T_0^0 Q_0^0 T_0^1 Q_1^0 T_3^2 I_3 d^T_{31}) = 0 \]
\[ h_{14} = h_{41} = Tr(T_0^0 Q_0^0 T_0^1 Q_1^0 T_4^4 I_4 d^T_{41}) = mL_4 [L_2C_3 + L_3C_2S_2S_3 + L_4C_2^2S_2S_3] \]

\[ h_{22} = Tr(T_1^0 Q_1^0 T_1^1 Q_2^1 T_2^1 I_2 d^T_{22}) = 0 \]
\[ h_{23} = h_{32} = Tr(T_1^0 Q_1^0 T_1^1 Q_2^1 T_3^2 I_3 d^T_{32}) = 0 \]
\[ h_{24} = h_{42} = -mL_4 \left[ C_3S_2(L_2 + L_3S_3) + L_5S_2C_3^2(C_1 + S_3) \right] \]
\[ h_{33} = Tr(T_3^0 Q_3^3 T_3^2 Q_3^2 T_4^4 I_4 d^T_{33}) = 0 \]
\[ h_{34} = h_{43} = Tr(T_1^0 Q_1^0 T_1^1 Q_2^1 T_4^4 I_4 d^T_{34}) = mL_4 S_2^2S_3 \left[ L_2 + L_3 \right] \]
\[ h_{44} = Tr(T_0^0 Q_0^0 T_0^1 Q_1^0 T_2^1 I_4 d^T_{44}) = 0 \]

\[ h_{21} = h_{22} = h_{31} = h_{32} = h_{33} = h_{34} = h_{41} = h_{42} = h_{43} = h_{44} = 0 \]

\[ Tr(T_0^0 Q_1^0 T_1^1 Q_2^1 T_2^1 I_4 d^T_{42}) = mL_4 C_3 \left[ L_2S_2 + L_4C_2S_3 \right] \]

\[ h_{224} = h_{242} = Tr(T_1^0 Q_1^0 T_1^1 Q_2^1 T_2^1 I_4 d^T_{42}) = h_{234} = h_{243} = Tr(T_2^0 Q_2^0 T_2^1 Q_3^1 T_4^4 I_4 d^T_{42}) = -mL_4 L_4 C_3 \]
\[ h_{244} = Tr(T_3^0 Q_3^3 T_3^2 Q_3^1 T_4^4 I_4 d^T_{42}) = -mL_4 L_4 C_3 \]

The Coriolis and centrifugal torque terms are computed using the series summation:
\[ H_{ij} = \sum_h q_h q_i \quad \text{for } i,j,k = 1,2,3,4 \]

Substituting the values of \( i, j, k \) in the above equation and simplifying gives
\[ H_1 = 2h_{114} \theta_1 \theta_4 + 2h_{124} \theta_2 \theta_4 + 2h_{144} \theta_4 \theta_4 + h_{144} \theta_4^2 \]
\[ H_3 = 2h_{314} \theta_3 \theta_3 + 2h_{324} \theta_3 \theta_4 + 2h_{344} \theta_4 \theta_4 + h_{344} \theta_4^2 \]
\[ H_4 = 0 \quad H_{ij} = 0 \]

The mass of the links is at the distal end of the links \( \{1\}, \{2\}, \{3\}, \{4\} \). Thus,
\[ r_1 = \left[ \begin{array}{c} 0 \\ 0 \\ L_4 \\ 1 \end{array} \right] \quad x_2 = \left[ \begin{array}{c} 0 \\ 0 \\ L_2 \\ 1 \end{array} \right] \]
And the gravity is in the positive direction of Z-axis of frame \( \{0\} \), that is, \( g = \left[ \begin{array}{ccc} 0 & 0 & -g \end{array} \right] \)

Where \( g = 9.802 \text{ m/s}^2 \). Therefore the gravity loading \( G_1 \) at the joint 1 is,
\[ G_1 = \left(m_1 gd_1 r_1^{-1} + m_2 gd_2 r_2^{-2} + m_3 gd_3 r_3^{-3} + m_4 gd_4 r_4^{-4} \right) \quad \therefore G_1 = 0 \]

Similarly gravity loading at the joint 2,3, and 4 is \( G_2 \), \( G_3 \) and \( G_4 \) respectively.
\[ G_2 = \left(m_1 L_2 g C_2 + 2m_2 L_3 g C_3 + m_3(gC_4(L_4 - L_3) - gL_5 S_3) \right) \quad \therefore G_3 = \left(m_4 L_4 g C_2 S_3 \right) \]
1.4 Power = Torque

Angular velocity = $\sum_{i=1}^{n} \omega_i$

Power= $\tau_1 \omega_1 + \tau_2 \omega_2 + \tau_3 \omega_3 = \tau_1 \dot{\theta}_1 + \tau_2 \dot{\theta}_2 + \tau_3 \dot{\theta}_3$

1.4 Closure

Study of basic knowledge of dynamics was carried out. The necessary equations of motions were studied to gain the knowledge necessary for deriving the equations for dynamic analysis. Some of the important algorithms are summarized. The advantages of Lagrange approach is stated which motivated me to work with Lagrange Euler approach. This approach is used to formulate the torque equation for lifting a given load for a given angle.

1.5 REFERENCES:


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